AP Calculus AB

Summer Preparation

Name ___________________
Topic #1: GRAPHING CALCULATOR SKILLS

All students are required to have a graphing calculator (GC) for use with our course. Our course is taught with the Texas Instrument TI-83/84 series. This includes the TI-83, TI-83 Plus, TI-84, and TI-84 Plus (including Silver Edition of these). Other makes and models are permitted, but the series listed will be used during instruction.

Students are expected to be joining AP Calculus with the knowledge of “intermediate” GC skills. These include:

- Graphing an equation
- Finding a suitable viewing Window
- Finding the Intersection of two curves
- Finding the Roots (Zeros, x-intercepts) of a curve
- Finding the Max/Min point on a curve
- Evaluating equations for a specified x-value

All of these are built-in features that you should be adept at using upon joining us this Fall. If your GC skills do not include these items, it would be in your best interest to learn them over the summer. Knowledge of these skills is assumed for entering AP Calculus.

To assist you with any skill-building that might be necessary, here are several websites that should be of help:

- Using the TI-89 Graphing Calculator http://www.prenhall.com/divisions/esm/app/graphing/ti89/
- TI-83/84 Tutorials by topic [another great site] http://calculator.maconstate.edu/calc_topics.html
- TI-83/84 Trouble Shooting http://oakroadsystems.com/math/ti83oops.htm

Problems:

1) Graphically solve \( 2 \sin x = x \). Round solutions to the nearest thousandth. (Note: Must be in radian mode when graphing trig functions)

2) Graph \( f(x) = 3xe^{-0.2x} \) on your GC. Zoom – 6 to graph. Round any decimals to the nearest thousandth.
   a. Domain?
   b. Range?
   c. Maximum Points (ordered pair)
   d. Increasing on what interval(s)?
   e. Decreasing on what interval(s)?
   f. Asymptotes?
Topic #2: SOLVING EQUATIONS

Should you need a refresher, check out the following websites:

- Just Math Tutoring (click on Algebra/SAT videos on the left and scroll down to “Solving equations and inequalities”) http://www.justmathtutoring.com/
- From the Univ. of Houston Online Mathematics http://online.math.uh.edu/Math1310/

Solve each of the following equations algebraically by hand. Feel free, though, to check your solutions graphically!

1)  \( \frac{2}{3}p = \frac{1}{2}p + \frac{1}{3} \)

2)  \( 1 - \frac{1}{2}x = 6 \)

3)  \( x^2 - 7x + 12 = 0 \)

4)  \( 3x^2 + 5x = 2 \)

5)  \( x^4 = x^2 \)

6)  \( x^2 + 25 = 10x \)

7)  \( \frac{x+2}{2} - \frac{3}{4} = x \)

8)  \( \frac{1}{a} + \frac{1}{2} = \frac{2}{a} \)

9)  \( (2x + 5)^{1/4} = 2 \)

10)  \( (x - 1)^3 = 27 \)

11)  \( \frac{x}{x-3} + \frac{1}{3} = 1 \)

12)  \( \sqrt{x} = x - 2 \)
# Topic #3: INTERVAL NOTATION

From the Univ. of Houston Online Mathematics Site...

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Description of Interval</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(b, \infty)$</td>
<td>includes all real numbers $x$ such that $x$ is greater than $b$ $(x &gt; b)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$[b, \infty)$</td>
<td>includes all real numbers $x$ such that $x$ is greater than or equal to $b$ $(x \geq b)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$(-\infty, a)$</td>
<td>includes all real numbers $x$ such that $x$ is less than $a$ $(x &lt; a)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$(-\infty, a]$</td>
<td>includes all real numbers $x$ such that $x$ is less than or equal to $a$ $(x \leq a)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$(-\infty, \infty)$</td>
<td>includes all real numbers $x$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$(a, b)$</td>
<td>includes all real numbers $x$ such that $x$ is between $a$ and $b$ $(a &lt; x &lt; b)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$[a, b)$</td>
<td>includes all real numbers $x$ such that $x$ is greater than or equal to $a$ and $x$ is less than $b$ $(a \leq x &lt; b)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$(a, b]$</td>
<td>includes all real numbers $x$ such that $x$ is greater than $a$ and $x$ is less than or equal to $b$ $(a &lt; x \leq b)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>includes all real numbers $x$ such that $x$ is between and including $a$ and $b$ $(a \leq x \leq b)$</td>
<td>![Graph]</td>
</tr>
</tbody>
</table>

Other websites:
- Flash presentation from Univ. of Houston [http://www.online.math.uh.edu/Math1314/Lsn0/Topic-3/Topic-3.htm](http://www.online.math.uh.edu/Math1314/Lsn0/Topic-3/Topic-3.htm)

## Solution

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1)$ $-2 &lt; x \leq 4$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$2)$ $[-1, 7)$</td>
<td>![Graph]</td>
</tr>
<tr>
<td>$3)$</td>
<td>![Graph]</td>
</tr>
</tbody>
</table>
Topic #4: **GRAPHS OF PARENT FUNCTIONS**

The following functions are the basic “parent” functions that you should know instantly. Actual points should be plotted (at least three) in the creation of these graphs. You should be able to produce these “on demand” and know them “in your head” without having to graph them on your GC.

1) \( y = x \)

   **Domain:**
   
   **Range:**

2) \( y = |x| \)

   **Domain:**
   
   **Range:**

3) \( y = x^2 \)

   **Domain:**
   
   **Range:**

4) \( y = \sqrt{x} \)

   **Domain:**
   
   **Range:**
5) \( y = x^3 \)

- **Domain:**
- **Range:**

6) \( y = 2^x \)

- **Domain:**
- **Range:**

7) \( y = \log_2 x \)

- **Domain:**
- **Range:**

8) \( y = x^{1/3} \)

- **Domain:**
- **Range:**
Topic #5: **EQUATIONS OF LINES**

<table>
<thead>
<tr>
<th>Slope intercept form:  $y = mx + b$</th>
<th>Vertical line:  $x = c$ (slope is undefined)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point-slope form:  $y - y_1 = m(x - x_1)$</td>
<td>Horizontal line:  $y = c$ (slope is 0)</td>
</tr>
<tr>
<td>Standard form:  $Ax + By = C$</td>
<td>Slope:  $m = \frac{y_2 - y_1}{x_2 - x_1}$</td>
</tr>
</tbody>
</table>

1) Write an equation of a line with slope 3 and y-intercept 5.

2) Write an equation of a line passing through the point (5, -3) and with an undefined slope.

3) Write an equation of the line passing through the point (-4, 2) with a slope of 0.

4) Use point-slope form of a linear equation to find an equation of the line passing through the point (6, 5) with a slope of 2/3.

5) Find an equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

6) Find an equation of a line perpendicular to the y-axis and passing through the point (4, 7).

7) Find an equation of a line passing through the points (-3, 6) and (1, 2).

8) Find an equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).
Topic #7: SIMPLIFYING COMPLEX FRACTIONS

When simplifying complex fractions, you want to eliminate the “little denominators” by multiplying through what would be the LCM (Least Common Multiple) of them. You end up multiplying the “big numerator” and the “big denominator” by that LCM.

For example:

1) \( \frac{1 + \frac{1}{a}}{\frac{2}{a^2} - 1} \)
   
   The LCM of the “little denominator” would be \( a^2 \). So multiply the entire big numerator as well as the entire “big denominator” by \( a^2 \). Don’t forget to distribute properly!
   
   \[
   \frac{\left(1 + \frac{1}{a}\right) a^2}{\left(\frac{2}{a^2} - 1\right) a^2} = \frac{a^2 + a}{2 - a^2}
   \]

2) \( \frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} \)
   
   The denominator you want to eliminate is simply \( x + 1 \)... so that’s what you should multiply through by. Again, don’t forget to distribute in the “big numerator” properly!
   
   \[
   \frac{\left(-7 - \frac{6}{x+1}\right) x + 1}{\left(\frac{5}{x+1}\right) x + 1} = \frac{-7x - 7 - 6}{5} = \frac{-7x - 13}{5}
   \]

3) \( \frac{-2 + \frac{3x}{x-4}}{\frac{5}{x-4}} \)
   
   The LCM of the “little denominator” would be \( x(x - 4) \) So multiply the entire “big numerator” as well as the entire “big denominator” by \( x(x - 4) \). Don’t forget to distribute properly!
   
   \[
   \frac{\left(-2 + \frac{3x}{x-4}\right) x(x - 4)}{\left(\frac{5}{x-4}\right) x(x - 4)} = \frac{-2x + 8 + 3x^2}{5x^2 - 20x - x} = \frac{3x^2 - 2x + 8}{5x^2 - 21x}
   \]

Websites should you need extra help:

- From Univ. of Houston (I like method #2 best) [http://online.math.uh.edu/Math1300/ch5/s54/index.html](http://online.math.uh.edu/Math1300/ch5/s54/index.html)
Simplify each of the following:

1) \[ \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} \]

2) \[ \frac{1 - \frac{1}{x^3}}{3 + \frac{1}{x^2}} \]

3) \[ \frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}} \]

4) \[ \frac{5 + \frac{1}{n} - \frac{6}{n^2}}{\frac{2}{n} - \frac{2}{n^2}} \]

5) \[ \frac{x - 1}{x + 1} - \frac{x}{x} \]

\[ \frac{1}{x+1} - \frac{1}{x} \]
### Topic #8: SOLVING SYSTEMS OF EQUATIONS

#### Solving by the Substitution Method:

1. Solve one equation for the variable with coefficient 1 or -1.
2. Substitute this into the other (unused) equation.
3. Solve for the variable.
4. Substitute found value into re-arranged equation to solve for value of 2\textsuperscript{nd} variable.

Websites should you need extra help: [http://www.purplemath.com/modules/systlin4.htm](http://www.purplemath.com/modules/systlin4.htm)

#### Solving by the Elimination Method:

1. Find opposite (positive vs. negative) coefficients for one of the variables.
2. Multiply equation(s) by constant(s) so as to obtain these opposite coefficients.
3. Add the two equations together so as to eliminate one of the variables.
4. Solve for the variable.
5. Substitute found value into re-arranged equation to solve for value of 2\textsuperscript{nd} variable.

Websites should you need extra help: [http://www.purplemath.com/modules/systlin5.htm](http://www.purplemath.com/modules/systlin5.htm)

Solve each system of equations by using either Substitution Method or Elimination Method.

1) \[
\begin{align*}
2x + y &= 4 \\
3x + 2y &= 1
\end{align*}
\]

2) \[
\begin{align*}
2x + y &= 4 \\
3x - y &= 14
\end{align*}
\]

3) \[
\begin{align*}
2x - 5y &= 13 \\
6x + 3y &= 10
\end{align*}
\]
Topic #9: USING THE LAW OF EXPONENTS

For #1-9, simplify each of the following by using the Laws of Exponents. There should be NO NEGATIVE EXPONENTS in your final simplifications.

1) \(4m(3a^2m)\)
2) \((2x^2)^{-1}\)
3) \((2x^{-1})^2\)
4) \((2x^{-2})(3x^{-5})\)
5) \(\frac{2xy^{-1}}{x^{-2}}\)
6) \(\frac{(n^3p)^2}{(np)^{-2}}\)
7) \(\frac{6x^{-2}y^3}{xy^4}\)
8) \(\frac{6x^{-2}y}{12x^2y^3}\)
9) \(\left(\frac{x}{y^2}\right)^{-2}(y^2)^{-1}\)

For #10-23, evaluate each of the following WITHOUT the use of a calculator! Problems such as these you should be able to do in your head!

10) \(2^{-2} \cdot 2^{-5}\)
11) \(4^{1/2}\)
12) \(4^{-1/2}\)
13) \(8^{1/3}\)
14) \((-8)^{1/3}\)
15) \(\left(\frac{1}{4}\right)^{-1/2}\)
16) \(\left(\frac{1}{4}\right)^{1/2}\)
17) \((8)^{-3}\)
18) \((8)^{1/3}\)
19) \(\left(\frac{1}{8}\right)^{1/3}\)
20) \((-4)^{1/2}\)
21) \(\left(\frac{1}{8}\right)^{-1/3}\)
22) \((2)^{1/2}\)
23) \(-4^{1/2}\)
Topic #10: COMPLETE FACTORIZATION OF POLYNOMIALS

Factorization of polynomials is one of those topics that shows up throughout AP Calculus. You should be able to do it “on demand” and as the need arises.

Factoring Patterns:

- Differences of Two Squares: \[ a^2 - b^2 = (a + b)(a - b) \]
- Sum of Two Cubes: \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
- Difference of Two Cubes: \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

Websites should you need extra help:

- From Univ. of Houston [http://online.math.uh.edu/Math1300/index.html](http://online.math.uh.edu/Math1300/index.html) Chapter 4
- And [http://online.math.uh.edu/Math1330/Appendix/sA1/index.html](http://online.math.uh.edu/Math1330/Appendix/sA1/index.html)

Completely factor each polynomial. Remember to first look for a GCF!!

1) \[ 3y^2 + 9y + 6 \]
2) \[ 3w^2 - 12w + 12 \]
3) \[ 16u^8 - 8u^4 + 1 \]
4) \[ 2t^2 - 28t + 98 \]
5) \[ 2a^3 - 2a \]
6) \[ 3k^2 + k - 2 \]
7) \[ 24p^2 - 54j^2 \]
8) \[ 2x^2 + 10x - 28 \]
9) \[ 12m^2 - 10m - 12 \]
10) \[ 6x^2 + 31x + 35 \]
11) $63p^2 + 11p - 2$

12) $15x^2 - 26x + 8$

13) $2f^3 - 20f^2 + 18f$

14) $25x^2 + 10xy + y^2$

15) $9c^2 + 30c + 25$

16) $e^4 - 9c^2$

17) $5x^3y^2 - 45x$

18) $64x^3 + y^3$

19) $8x^3 - 27y^3$

20) $216a^3 - 1$

21) $125x^3 + y^3$

22) $9x(3x - 9)^2 + (3x - 9)^3$

23) $(x + 1)^{3/2} + (x + 1)^{1/2}$

24) $2x(x - 1)^{-1/2} + 5x(x - 1)^{1/2}$

25) $-2x^4(x^4 + 4)^{-3/2} + (x^4 + 4)^{-1/2}$
Topic #11: FUNCTION COMPOSITION & INVERSES

Composition of Functions:

There are two notations for the composition of functions: \( f[g(x)] \) or \((f \circ g)(x)\). Each is read as “\( f \) of \( g \) of \( x \)” or “\( f \) compose \( g \) of \( x \)”.

Functions are composed by substituting the second function in place of the variable in the first function.

For example, if \( f(x) = 2x^2 + 1 \) and \( g(x) = x - 4 \).

\[
f(g(x)) = f(x - 4) = 2(x - 4)^2 + 1 = 2(x^2 - 8x + 16) + 1 = 2x^2 - 16x + 33
\]

Websites should you need extra help:
- YouTube “Composition of Functions”
- From Univ. of Houston [http://online.math.uh.edu/Math1330/ch1/s14/index.html](http://online.math.uh.edu/Math1330/ch1/s14/index.html)

Inverses of Functions:

To find the inverse \( f^{-1}(x) \) of a function \( f(x) \), switch the \( x \) and \( y \), then solve the new equation for \( y \).

For example, find the inverse of \( f(x) = \frac{1}{\sqrt{x} + 1} \).

\[
y = \frac{1}{\sqrt{x} + 1}
\]

\[
x = \frac{1}{y + 1}
\]

\[
x^3 = y + 1
\]

\[
f^{-1}(x) = x^3 - 1
\]

Websites should you need extra help:
- YouTube “Finding an Inverse Function”
- From Univ. of Houston [http://online.math.uh.edu/Math1330/ch1/s15/index.html](http://online.math.uh.edu/Math1330/ch1/s15/index.html)

For #1-2, given \( f(x) \) and \( g(x) \), find both \( f \circ g \) and \( g \circ f \).

1) \( f(x) = x - 1 \) and \( g(x) = 2x^2 \)

2) \( f(x) = 3x - 2 \) and \( g(x) = x^2 - 4 \)
For #3-5, determine the inverse of each function given below.

3) \( f(x) = x^3 - 8 \)

4) \( f(x) = \frac{2}{3+x} \)

5) \( f(x) = \frac{1}{2}x - \frac{1}{3} \)
Topic #12: LOGARITHMS

Evaluating Logarithms:

Examples:

- \( \log_2 8 = 3 \) because \( 2^3 = 8 \)
- \( \log_{16} 4 = \frac{1}{2} \) because \( 16^{1/2} = \sqrt{16} = 4 \)
- \( \log_2 \left( \frac{1}{8} \right) = -3 \) because \( 2^{-3} = \frac{1}{2^3} = \frac{1}{8} \)
- \( \ln e^3 = 3 \) because a Natural Logarithm is just a logarithm in base \( e \).

The Laws of Logarithms:

For all \( a > 0 \) (\( a \neq 1 \)), \( n \in R \) (is a real number), and \( u, v \in R^+ \) (\( u \) and \( v \) are positive real numbers)

1. \( \log_a (uv) = \log_a u + \log_a v \)
2. \( \log_a \left( \frac{u}{v} \right) = \log_a u - \log_a v \)
3. \( \log_a u^n = n \log_a u \)
4. \( \log_a u = \log_a v \) if and only if \( u = v \)

Websites should you need extra help:

- From Univ. of Houston [http://online.math.uh.edu/Math1330/ch3/s33/index.html](http://online.math.uh.edu/Math1330/ch3/s33/index.html)
- Just Math Tutoring [www.justmathtutoring.com](http://www.justmathtutoring.com) [click on Algebra/SAT Videos on the left and scroll down the page to the three links for logarithms]

For #1-4, evaluate each logarithm “in your head” without the use of a GC!

1) \( \log_4 64 = \) 3) \( \ln e^6 = \)

2) \( \log_{10} 3 = \) 4) \( \log_3 \frac{1}{27} = \)

5) Express \( 3^5 = 243 \) as a logarithm.

6) Express \( e^{-3} \approx 0.0498 \) as a logarithm.

7) Express \( \log_5 27 = 2 \) in exponential form.
For #8-11, write each logarithm in expanded form.

8) \( \log_b (xy)^3 \)

9) \( \log_4 \frac{a^5}{64} \)

10) \( \log_b \frac{8r^2}{\sqrt{b}} \)

11) \( \log_2 \frac{16x}{8y^3} \)

For #12-15, write each expression as a logarithm of one single expression.

12) \( 2 \log_3 x + 4 \log_3 y \)

13) \( 4 \log_b t - 3 \log_b v \)

14) \( \log_5 6c^4 - \log_5 2c \)

15) \( \log_2 3t + \log_2 5t \)
Topic #13: TRIGONOMETRY

The beauty of using the Unit Circle is that there are really only 7 ordered pairs you need to know! These are marked for you in the circle to the left. Using these in conjunction with Reference Angles will enable you to find any trig value you might be asked:

\[
\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \\
\tan \frac{\pi}{4} = 1 \\
\tan \frac{\pi}{3} = \sqrt{3}
\]

Using the Unit Circle and Reference Angles to evaluate trig expressions:

For Example:

1. Find \(\sin \frac{11\pi}{6}\). The terminal side of this angle lies in Q IV, so the sine value will be negative. Reference angle = \(\frac{\pi}{6}\) whose sine is \(\frac{1}{2}\). Therefore, \(\sin \frac{11\pi}{6} = -\frac{1}{2}\).

2. Find \(\sec \left( -\frac{4\pi}{3} \right)\). The terminal side of this angle lies in Q II, so the secant value (reciprocal of cosine) will be negative. Reference angle = \(\frac{\pi}{3}\) and \(\cos \frac{\pi}{3} = \frac{1}{2}\). So \(\sec \left( -\frac{4\pi}{3} \right) = -2\).

Websites should you need extra help:

- From Jim Rahn, author, consultant, and math teacher in New Jersey
- Just Math Tutoring [www.justmath tutoring.com](http://www.justmath tutoring.com) and click on “Free Trigonometry Videos” on left.
- From Univ. of Houston [http://online.math.uh.edu/Math1330/ch4/s43/index.html](http://online.math.uh.edu/Math1330/ch4/s43/index.html)

Convert the following from degrees to radians or from radians to degrees.

1) \(-135^\circ\)  
2) \(135^\circ\)  
3) \(210^\circ\)  
4) \(\frac{5\pi}{4}\)  
5) \(\frac{4\pi}{3}\)
For #6-32, evaluate each trigonometric expression WITHOUT using a GC. You should be capable of doing these quickly “in your head”!

6) \( \sin \frac{\pi}{4} \)  
15) \( \sec \frac{\pi}{2} \)  
24) \( \csc \left( -\frac{2\pi}{3} \right) \)  

7) \( \cos \frac{\pi}{3} \)  
16) \( \sin \frac{7\pi}{6} \)  
25) \( \sec \frac{\pi}{2} \)  

8) \( \sin \frac{3\pi}{2} \)  
17) \( \cot \frac{\pi}{2} \)  
26) \( \cot \pi \)  

9) \( \tan \frac{2\pi}{3} \)  
18) \( \tan \frac{4\pi}{3} \)  
27) \( \sin \frac{5\pi}{4} \)  

10) \( \sin \frac{\pi}{3} \)  
19) \( \cot \frac{5\pi}{6} \)  
28) \( \cos \left( -\frac{\pi}{6} \right) \)  

11) \( \tan \left( -\frac{\pi}{2} \right) \)  
20) \( \tan \frac{\pi}{2} \)  
29) \( \csc \frac{\pi}{6} \)  

12) \( \tan \frac{3\pi}{4} \)  
21) \( \sec \frac{7\pi}{4} \)  
30) \( \tan \frac{5\pi}{3} \)  

13) \( \cos \left( -\frac{2\pi}{3} \right) \)  
22) \( \cos \frac{5\pi}{6} \)  
31) \( \cos \left( -\frac{11\pi}{6} \right) \)  

14) \( \csc \frac{\pi}{3} \)  
23) \( \cos \frac{\pi}{2} \)  
32) \( \cot \left( -\frac{\pi}{6} \right) \)
Topic #14: SOLVING TRIG EQUATIONS

Method of solution:

1) Isolate the trig function to one side of the equation.
2) Determine the reference angle.
3) Determine the quadrant in which the answer(s) will lie depending upon if the trig value is positive or negative.
4) Find the final angle answer(s) – one for each of the quadrants determined above.

For Example:

1. Find all solutions in \([0, 2\pi]\) of \(\sqrt{2} \sin x = -1\). Rewrite as \(\sin x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}\). From the Unit Circle, the reference angle is \(\frac{\pi}{4}\). Since the sine value is to be negative, one angle answer will lie in QIII and the other will lie in QIV. The QIII answer is \(\frac{5\pi}{4}\) and the QIV answer is \(\frac{7\pi}{4}\).

2. Find all solutions in \([0, 2\pi]\) of \(2 \cos 2x = \sqrt{3}\). Rewrite this as \(\cos 2x = \frac{\sqrt{3}}{2}\). From the Unit Circle, the reference angle is \(\frac{\pi}{6}\). Since the cosine value is to be positive, one angle answer will lie in QI and the other in QIV. Solve \(2x = \frac{\pi}{6}\) and \(2x = \frac{11\pi}{6}\) to find that \(x = \frac{\pi}{12}\) and \(x = \frac{11\pi}{12}\).

Website should you need extra help:

- From Univ. of Houston [http://online.math.uh.edu/Math1330/ch6/s63/index.html](http://online.math.uh.edu/Math1330/ch6/s63/index.html)
- You Tube “Solving Trig Equations”

Solve each equation on the interval \([0, 2\pi]\). All answers are to be in terms of \(\pi\)!

1) \(\sin x = -\frac{1}{2}\) 
5) \(\sin^2 x = \frac{1}{2}\)

2) \(2 \cos x = \sqrt{3}\) 
6) \(\sin 2x = -\frac{\sqrt{3}}{2}\)

3) \(\sqrt{3} \tan x = 1\) 
7) \(\cos 2x = -\frac{1}{\sqrt{2}}\)

4) \(\tan x = -\sqrt{3}\)
# Topic #15: INVERSE TRIG FUNCTIONS

Evaluating inverse trig functions is like playing Jeopardy. Instead of being given the angle and asked for the sine, cosine, or tangent value (or any of their reciprocals), you are given the trig value and have to find the angle.

Remember there are two different notations for inverse trig functions:

- \( \arcsin x \) OR \( \sin^{-1} x \)
- \( \arccos x \) OR \( \cos^{-1} x \)
- \( \arctan x \) OR \( \tan^{-1} x \)

Similar notation follows for their reciprocal trig functions.

For example: “Find \( \arcsin \frac{1}{2} \)” means to find the angle whose sine is \( \frac{1}{2} \).

The trick, though, is that since these are FUNCTIONS, there can only be NOE angle answer for each. So there are “restricted ranges” from which your answers will come. You need to be good at using the Unit Circle in order to do these easily!

<table>
<thead>
<tr>
<th>Restricted range</th>
<th>Angle solution will be in Quadrant…</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \sin^{-1} x ) ([-\pi/2, \pi/2])</td>
<td>Q IV or I</td>
</tr>
<tr>
<td>( y = \cos^{-1} x ) ([0, \pi])</td>
<td>Q I or II</td>
</tr>
<tr>
<td>( y = \tan^{-1} x ) ([-\pi/2, \pi/2])</td>
<td>Q IV or I</td>
</tr>
</tbody>
</table>

Examples...

1. Find \( \cos^{-1} \frac{1}{2} \). This means to find an angle whose cosine is \( \frac{1}{2} \). There are many angles that exist, but because of the restricted range, the one angle answer will lie in either Q I (where cosine is positive) or Q II (where cosine is negative). So the only angle answer is the positive value, and \( \cos^{-1} \frac{1}{2} = \frac{\pi}{3} \).

2. Find \( \arctan (-\sqrt{3}) \). This means to find an angle whose tangent is \( -\sqrt{3} \). There are many angles that exist, but because of the restricted range, the one angle answer will lie in either Q IV (where tangent is negative) or Q I (where tangent is positive). So \( \arctan (-\sqrt{3}) = -\frac{\pi}{3} \).

3. Find \( \sec^{-1} \left( -\frac{2}{\sqrt{3}} \right) \). This means to find an angle whose secant is \( -\frac{2}{\sqrt{3}} \), or rather the cosine value is \( -\frac{\sqrt{3}}{2} \).

There are many angles that exist, but because of the restricted range, the one angle answer will lie in either Q I (where cosine is positive) or Q II (where cosine is negative). So \( \sec^{-1} \left( -\frac{2}{\sqrt{3}} \right) = \cos^{-1} \left( -\frac{\sqrt{3}}{2} \right) = \frac{5\pi}{6} \).

Websites should you need extra help:

- From Univ. of Houston [http://online.math.uh.edu/Math1330/ch5/s54/index.html](http://online.math.uh.edu/Math1330/ch5/s54/index.html)
- You Tube “Inverse Trig Functions”
Evaluate each inverse trig expression. Note: some values may not exist.

1) \( \sin^{-1} \frac{1}{2} \)

2) \( \cos^{-1} \frac{1}{2} \)

3) \( \arctan \frac{\sqrt{3}}{3} \)

4) \( \arccos \left( -\frac{\sqrt{3}}{2} \right) \)

5) \( \tan^{-1} -\sqrt{3} \)

6) \( \cos^{-1} -\frac{1}{2} \)

7) \( \arcsin \frac{\sqrt{3}}{2} \)

8) \( \sin^{-1} 0 \)

9) \( \arccos 1 \)

10) \( \arctan 0 \)

11) \( \arcsin 2 \)

12) \( \cos^{-1} (-1) \)

13) \( \cos^{-1} \left( -\frac{\sqrt{2}}{2} \right) \)

14) \( \sin^{-1} \left( -\frac{\sqrt{2}}{2} \right) \)

15) \( \arctan (-1) \)