3-5 Study Guide and Intervention

Arithmetic Sequences as Linear Functions

Recognize Arithmetic Sequences
A **sequence** is a set of numbers in a specific order. If the difference between successive terms is constant, then the sequence is called an **arithmetic sequence**.

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Arithmetic Sequence	a numerical pattern that increases or decreases at a constant rate or value called the **common difference**
Terms of an Arithmetic Sequence	If a_1 is the first term of an arithmetic sequence with common difference d, then the sequence is $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$
nth Term of an Arithmetic Sequence	$a_n = a_1 + (n - 1)d$

Example 1 Determine whether the sequence $1, 3, 5, 7, 9, 11, \ldots$ is an arithmetic sequence. Justify your answer.

If possible, find the common difference between the terms. Since $3 - 1 = 2$, $5 - 3 = 2$, and so on, the common difference is 2.

Since the difference between the terms of $1, 3, 5, 7, 9, 11, \ldots$ is constant, this is an arithmetic sequence.

Example 2 Write an equation for the nth term of the sequence $12, 15, 18, 21, \ldots$.

In this sequence, a_1 is 12. Find the common difference.

$$12 \quad 15 \quad 18 \quad 21$$
$$+3 \quad +3 \quad +3$$
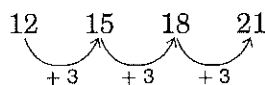
The common difference is 3.

Use the formula for the nth term to write an equation.

$a_n = a_1 + (n - 1)d$ Formula for the nth term
$a_n = 12 + (n - 1)3$ $a_1 = 12, d = 3$
$a_n = 12 + 3n - 3$ Distributive Property
$a_n = 3n + 9$ Simplify.

The equation for the nth term is $a_n = 3n + 9$.

Exercises

Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain.

1. $1, 5, 9, 13, 17, \ldots$
  +4 +4 +4 +4
  Yes, d = 4

2. $8, 4, 0, -4, -8, \ldots$
  -4 -4 -4 -4
  Yes, d = -4

3. $1, 3, 9, 27, 81, \ldots$
  +2 +6 +18 +54
  No, no common difference

Find the next three terms of each arithmetic sequence.

4. $9, 13, 17, 21, 25, \ldots$
  +4 +4 +4 +4
  29, 33, 37

5. $4, 0, -4, -8, -12, \ldots$
  -4 -4 -4 -4
  -16, -20, -24

6. $29, 35, 41, 47, \ldots$
  +6 +6 +6
  53, 59, 65

Write an equation for the nth term of each arithmetic sequence. ~~Then graph the first five terms of the sequence.~~

7. $1, 3, 5, 7, \ldots$
  +2 +2 +2
  $a_n = 1 + (n-1)(2)$

8. $-1, -4, -7, -10, \ldots$
  -3 -3 -3
  $a_n = -1 + (n-1)(-3)$

9. $-4, -9, -14, -19, \ldots$
  -5 -5 -5
  $a_n = -4 + (n-1)(-5)$

3-5 Study Guide and Intervention (continued)

Arithmetic Sequences as Linear Functions

Arithmetic Sequences and Functions An **arithmetic sequence** is a linear function in which n is the independent variable, a_n is the dependent variable, and the common difference d is the slope. The formula can be rewritten as the function $a_n = a_1 + (n - 1)d$, where n is a counting number.

Example SEATING There are 20 seats in the first row of the balcony of the auditorium. There are 22 seats in the second row, and 24 seats in the third row.

a. Write a function to represent this sequence.

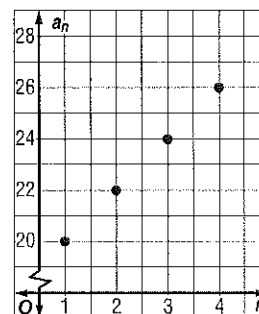The first term a_1 is 20. Find the common difference.
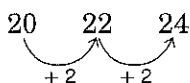
$$20 \quad 22 \quad 24$$
$$\quad +2 \quad +2$$

The common difference is 2.

$a_n = a_1 + (n - 1)d$ Formula for the nth term

$\quad = 20 + (n - 1)2$ $a_1 = 20$ and $d = 2$

$\quad = 20 + 2n - 2$ Distributive Property

$\quad = 18 + 2n$ Simplify.

The function is $a_n = 18 + 2n$.

b. Graph the function.

The rate of change is 2. Make a table and plot points.

n	a_n
1	20
2	22
3	24
4	26

Exercises

1. **KNITTING** Sarah learns to knit from her grandmother. Two days ago, she measured the length of the scarf she is knitting to be 13 inches. Yesterday, she measured the length of the scarf to be 15.5 inches. Today it measures 18 inches. Write a function to represent the arithmetic sequence.

$d = 2.5$

$a_n = 13 + (n-1)(2.5)$

2. **REFRESHMENTS** You agree to pour water into the cups for the Booster Club at a football game. The pitcher contains 64 ounces of water when you begin. After you have filled 8 cups, the pitcher is empty and must be refilled.

    a. Write a function to represent the arithmetic sequence.

    $a_n = 64 + (n-1)(-8)$

    b. Graph the function.

Lesson 3-5