

# 7-6 Study Guide and Intervention

## Growth and Decay

**Exponential Growth** Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

<b>Exponential Growth</b>	<p>The general equation for exponential growth is <math>y = a(1 + r)^t</math>.</p> <ul style="list-style-type: none"> <li>• <math>y</math> represents the final amount.</li> <li>• <math>a</math> represents the initial amount.</li> <li>• <math>r</math> represents the rate of change expressed as a decimal.</li> <li>• <math>t</math> represents time.</li> </ul>
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**Example 1: POPULATION** The population of Johnson City in 2005 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.

a. Write an equation to represent the population of Johnson City since 2005.

The rate 3.2% can be written as 0.032.

$$y = a(1 + r)^t$$

$$y = 25,000(1 + 0.032)^t$$

$$y = 25,000(1.032)^t$$

b. According to the equation, what will the population of Johnson City be in 2015?

In 2015  $t$  will equal 2015 - 2005 or 10. Substitute 10 for  $t$  in the equation from part a.

$$y = 25,000(1.032)^{10} \quad t = 10$$

$$\approx 34,256$$

In 2015 the population of Johnson City will be about 34,256.

**Example 2: INVESTMENT** The Garcias have \$12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.

The rate 3.5% can be written as 0.035. The special equation for compound interest is  $A = P \left(1 + \frac{r}{n}\right)^{nt}$ , where  $A$  represents the balance,  $P$  is the initial amount,  $r$  represents the annual rate expressed as a decimal,  $n$  represents the number of times the interest is compounded each year, and  $t$  represents the number of years the money is invested.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$= 12,000 \left(1 + \frac{0.035}{12}\right)^{3(12)}$$

$$\approx 13,326.49$$

In three years, the balance of the account will be \$13,326.49.

### Exercises

1. **POPULATION** The population of the United States has been increasing at an average rate of 0.91%. If the population was about 303,146,000 in 2008, predict the population in 2012.

About 314,332,051

3. **POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the 2008 population was about 6,641,000,000, predict the 2015 population.

About 7,269,417,259

2. **INVESTMENT** Determine the value of an investment of \$2500 if it is invested at an interest rate of 5.25% compounded monthly for 4 years.

\$3082.78

4. **INVESTMENT** Determine the value of an investment of \$100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.

\$185,888.87

$$1.) \quad y = 303,146,000(1+0.0091)^4$$

$$y = 314,332,051$$

$$2.) \quad A = 2500 \left(1 + \frac{0.0525}{12}\right)^{12(4)}$$

$$A = 3082.78$$

$$3.) \quad y = 6,641,000,000(1+0.013)^7$$

$$y = 7,269,417,259$$

$$4.) \quad A = 100,000 \left(1 + \frac{0.052}{4}\right)^{4(12)}$$

$$A = 185,888.8657$$

# 7-6 Study Guide and Intervention *(continued)*

## Growth and Decay

<b>Exponential Decay</b>	<p>The general equation for exponential decay is <math>y = a(1 - r)^t</math>.</p> <ul style="list-style-type: none"> <li>• <math>y</math> represents the final amount.</li> <li>• <math>a</math> represents the initial amount.</li> <li>• <math>r</math> represents the rate of decay expressed as a decimal.</li> <li>• <math>t</math> represents time.</li> </ul>
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**Example: DEPRECIATION** The original price of a tractor was \$45,000. The value of the tractor decreases at a steady rate of 12% per year.

**a. Write an equation to represent the value of the tractor since it was purchased.**

The rate 12% can be written as 0.12.

$y = a(1 - r)^t$	General equation for exponential decay
$y = 45,000(1 - 0.12)^t$	$a = 45,000$ and $r = 0.12$
$y = 45,000(0.88)^t$	Simplify.

**b. What is the value of the tractor in 5 years?**

$y = 45,000(0.88)^t$	Equation for decay from part a
$y = 45,000(0.88)^5$	$t = 5$
$y \approx 23,747.94$	Use a calculator.

In 5 years, the tractor will be worth about \$23,747.94.

### Exercises

**1. POPULATION** The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2005, predict its population in the year 2015.

*About 6,813,204 People*

**2. DEPRECIATION** Mr. Gossell is a machinist. He bought some new machinery for about \$125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest \$100) at the end of 10 years?

*About \$24,600*

**3. ARCHAEOLOGY** The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of carbon-14 of 100 grams.

a. If the organism lived 5730 years ago, what is the concentration of carbon-14 today? *50 grams*

b. If the organism lived 11,460 years ago, determine the concentration of carbon-14 today. *25 grams*

**4. DEPRECIATION** A new car costs \$32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years.

*About \$16,243*

$$1.) y = a(1-r)^t$$

$$y = 7,450,349(1-0.0089)^{10}$$

$$y = 6,813,203.776$$

About 6,813,204 people

$$2.) y = a(1-r)^t$$

$$y = 125,000(1-0.15)^{10}$$

$$y = 24,609.3$$

About \$24,600

$$3.) a. \frac{100}{2} = 50 \text{ grams}$$

$$b. \frac{50}{2} = 25 \text{ grams}$$

$$4.) y = a(1-r)^t$$

$$y = 32,000(1-0.12)^4$$

$$y = 19,190.25152$$

$$y = 19,190(1-0.08)^2$$

$$y = 16,242.6$$

About \$16,243