Name Answer Key Date________ Class ________

LESSON 6-6

Reteach

Fundamental Theorem of Algebra

If r is a root of a polynomial function, then $(x - r)$ is a factor of the polynomial, $P(x)$. So, you can use the roots to write the simplest form of a polynomial function.

Write the simplest polynomial function with roots −4, −2, and 3.

Step 1 Write the factors of the polynomial, $P(x) = 0$.

$(x + 4)(x + 2)(x - 3) = 0$

Root (a)	−4	−2	3
Factor ($x - a$)	$x + 4$	$x + 2$	$x - 3$

Step 2 Multiply the first two factors, $(x + 4)(x + 2)$.

$(x^2 + 6x + 8)(x - 3) = 0$

Step 3 Multiply $(x^2 + 6x + 8)(x - 3)$. Then simplify.

$x^3 - 3x^2 + 6x^2 - 18x + 8x - 24 = 0$

$x^3 + 3x^2 - 10x - 24 = 0$

The function is $P(x) = x^3 + 3x^2 - 10x - 24 = 0$.

Write the simplest polynomial function with the given roots.

1. −5, 1, and 2

$(x + 5)(x - 1)(x - 2) = 0$

$(x^2 + 4x - 5)(x - 2) = 0$

$x^3 + 4x^2 - 5x - 2x^2 - 8x + 10 = 0$

$P(x) = x^3 + 2x^2 - 13x + 10 = 0$

2. −3, −1, and 0

$x(x + 3)(x + 1) = 0$

$x(x^2 + 4x + 3) = 0$

$P(x) = x^3 + 4x^2 + 3x = 0$

3. 1, 4, and 5

$(x-1)(x-4)(x-5)$

$(x^2 - 5x + 4)(x-5)$

$x^3 - 5x^2 + 4x - 5x^2 + 25x - 20$

$P(x) = x^3 - 10x^2 + 29x - 20 = 0$

4. −2, 3, and 6 $(x+2)(x-3)(x-6)$

$(x^2 - x - 6)(x-6)$

$x^3 - x^2 - 6x - 6x^2 + 6x + 36$

$P(x) = x^3 - 7x^2 + 36$

5. 2, 4, and 6 $(x-2)(x-4)(x-6)$

$(x^2 - 6x + 8)(x-6)$

$x^3 - 6x^2 + 8x - 6x^2 + 36x - 48$

$P(x) = x^3 - 12x^2 + 44x - 48$

6. −5, 0, and 5 $x(x+5)(x-5)$

$x(x^2 - 25) = 0$

$P(x) = x^3 - 25x = 0$

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LESSON 6-6

Reteach

Fundamental Theorem of Algebra (continued)

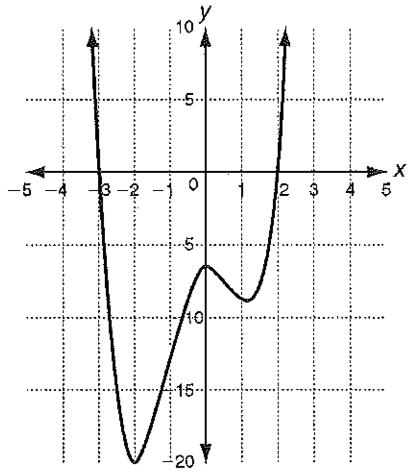
To solve $x^4 + x^3 - 5x^2 + x - 6 = 0$ means to find all the roots of the equation. A fourth degree equation has 4 roots.

Step 1 Identify possible real roots.

Possible roots, $\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm6$

The factors, p, of -6 are $\pm1, \pm2, \pm3, \pm6$. The factors, q, of 1 are ±1.

Step 2 Graph $y = x^4 + x^3 - 5x^2 + x - 6$.



Step 3 Test 2 as a root using synthetic substitution.

$$\begin{array}{r|rrrrr} 2 & 1 & 1 & -5 & 1 & -6 \\ & & 2 & 6 & 2 & 6 \\ \hline & 1 & 3 & 1 & 3 & \boxed{0} \end{array}$$

The remainder is 0, so 2 is a root. $(x - 2)(x^3 + 3x^2 + x + 3) = 0$

Test -3 as a root using synthetic substitution.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

The remainder is 0, so -3 is a root. $(x - 2)(x + 3)(x^2 + 1) = 0$

Step 4 Find the remaining roots.

$x^2 + 1 = 0$

$x = \pm i$

The roots of the equation are 2, -3, i, and $-i$.

Find the roots of the equation $x^4 - 3x^3 + 6x^2 - 12x + 8 = 0$.

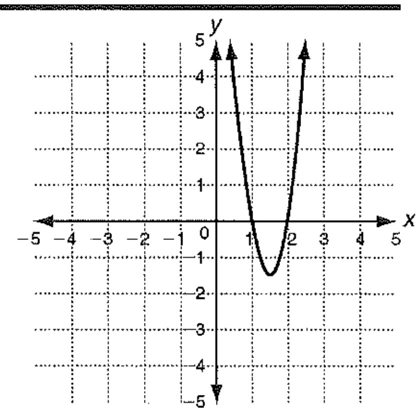
7. Possible roots: $\pm1, \pm2, \pm4, \pm8$

   Test: $x=1$ and $x=2$.

   1 and 2 are real roots.

   Solve x^2+4 to find the remaining roots.

   Remaining roots: $2i$ and $-2i$.



7) $x^4 - 3x^3 + 6x^2 - 12x + 8 = 0$

1	1	-3	6	-12	8
	↓	1	-2	4	-8
	1	-2	4	-8	0

$(x-1)(x^3 - 2x^2 + 4x - 8) = 0$

2	1	-2	4	-8
	↓	2	0	8
	1	0	4	0

$(x-1)(x-2)(x^2+4) = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$\sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm 2i}$