**3.1 Symmetry and Coordinate Graphs**

**Point Symmetry**

- Rotating a figure that has point symmetry $180^\circ$ should look the same.

![Graph showing point symmetry](image1)

**Point Symmetry**

- 2 points are symmetric with respect to (wrt) a 3rd point "M" if & only if M is the midpoint of the 2 points.

![Graph showing point symmetry](image2)
Origin Symmetry

- The graph of a relation $S$ is symmetric WRT the origin if and only if $(a,b) \in S$ implies that $(-a, -b) \in S$.

Algebraic test for determining point symmetry about the origin:

Determine whether the graph is symmetric WRT the origin.

*Example:* \( f(x) = x^2 \)

Your turn...

a) \( f(x) = x^6 \)

b) \( g(x) = -3x^3 + 5x \)
Line Symmetry

Four cases we will look at:
- \( \text{wrt } x\text{-axis} \)
- \( \text{wrt } y\text{-axis} \)
- \( \text{wrt } y=x \)
- \( \text{Wrt } y=-x \)

Line Symmetry  \( \text{wrt } x\text{-axis} \)

- If \((a, -b) \in S \text{ and } (a, b) \in S\)
- Substituting \((a, b)\) and \((a, -b)\) into the equation produces the same result.
Remember: For every point \((a, b)\), ________ is also on the graph.

Give examples of pairs of points that demonstrate \(x\)-axis symmetry.

**Line Symmetry wrt \(x\)-axis**

- Example: \(x = y^2\)

**Line Symmetry- wrt \(y\)-axis**

- If \((-a, b) \in S\) and \((a, b) \in S\)
- Substituting \((-a, b)\) and \((a, b)\) into the equation produces the same result.
Remember: For every point \((a, b)\), \((b, a)\) is also on the graph.

Give examples of pairs of points that demonstrate y-axis symmetry.

**Line Symmetry- wrt y-axis**

- Example: \(y = x^2 - 2\)

**Line Symmetry- wrt y=x**

- If \((b, a) \in S\) and \((a, b) \in S\)
- Substituting \((b, a)\) and \((a, b)\) into the equation produces the same result.
Line Symmetry - wrt $y=x$

- Example: $xy = 12$

$(2,6)$, $(6,2)$ are both members of this set

Line Symmetry - wrt $y=-x$

- If $(-b,-a) \in S$ and $(a, b) \in S$
- Substituting $(-b,-a)$ and $(a, b)$ into the equation produces the same result.
Line Symmetry - wrt $y=x$

- Example: $xy = -12$

$(2, -6), (-6, 2)$ are both members of this set

Practice

- Determine whether the graph of $x^2 + y = 3$ is symmetric with respect to the x-axis, the y-axis, the line $y = x$, the line $y = -x$. 
Practice:

- Can the graph of an equation be symmetric to more than one of these lines at a time? If yes, give an example, if no, give an explanation.

Practice

- Determine whether the graph of \(|y| = |x| + 1\) is symmetric with respect to the x-axis, the y-axis, both or neither. Use the information about the equation's symmetry to graph the relation.
Even and Odd Functions

- Even functions have symmetry wrt the y-axis
- Odd functions have symmetry wrt the origin

OOO YES!!
Practice:

Determine algebraically whether the functions are even, odd, or neither.

\[ f(x) = x^2 \]
\[ f(x) = x^6 \]
\[ g(x) = -3x^3 + 5x \]
\[ x = y^2 \]
\[ y = x^2 - 2 \]
\[ xy = 12 \]
\[ xy = -12 \]
Even and Odd Functions Summary

Even functions
1. Definition: if \( f(-x) = \) ____________ , then \( f(x) \) is an even function

2) Graph:
   a. The graph of an even function is symmetric about the ________________.
   b. If \((a, b)\) is on the graph, so is ____________.

3) Example of an even function:
   a. Graph \( f(x) = x^2 \)
      \[ \begin{array}{c|ccccccc}
          x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
          y & 9 & 4 & 1 & 0 & 1 & 4 & 9 \\
        \end{array} \]
   b. The points \((2, 4)\) and \((\ , \)\) are on the graph
   c. \( f(-x) = \)

Even and Odd Functions Summary

Odd functions
1. Definition: if \( f(-x) = \) ____________ , then \( f(x) \) is an odd function

2) Graph:
   a. The graph of an odd function is symmetric about the ________________.
   b. If \((a, b)\) is on the graph, so is ____________.

3) Example of an odd function:
   a. Graph \( f(x) = 3x \)
      \[ \begin{array}{c|ccccccc}
          x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
          y & -9 & -6 & -3 & 0 & 3 & 6 & 9 \\
        \end{array} \]
   b. The points \((2, 6)\) and \((\ , \)\) are on the graph
   c. \( f(-x) = \)
Homework

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& Worksheet
Symmetry

1. If a function is even, its graph is symmetric with respect to the ___________.
   This also means that \( f(-x) = \) ______.

2. If a function is odd, its graph is symmetric with respect to the ___________.
   This also means that \( f(-x) = \) ______.

Determine whether each function graphed is even, odd, or neither

Precalculus

Determine algebraically whether each of the following functions is even, odd or neither.

12. \( f(x) = 4x + 5 \)
13. \( f(x) = x^3 - x \)

14. \( f(x) = x^2 - 6 \)
15. \( f(x) = x^3 - x - 2 \)

16. \( f(x) = \frac{x^4 - x}{x^3 - x} \)
17. \( f(x) = \frac{x^4 - x}{x^3} \)

18. \( f(x) = (x - 4)^2 \)
19. \( f(x) = x^4 - x^3 + 4 \)
### Odd-Even Functions Reference

#### Odd Function Graphs

- The graphs of odd functions have rotational symmetry about the origin.

\[
y = x^3
\]

\[
y = \sin(x)
\]

#### Odd Function Table Values

<table>
<thead>
<tr>
<th>x</th>
<th>y = \sin(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-(\pi/2)</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\pi/2)</td>
<td>0</td>
</tr>
</tbody>
</table>

#### Odd Function Notation

If function f is an odd function and \(f(0) = 0\), then what is \(f(-x)\)?

<table>
<thead>
<tr>
<th>x</th>
<th>f(x) = f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
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<table>
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#### Even Function Graphs

- The graphs of even functions have y-axis symmetry.

\[
y = x^2
\]

\[
y = \cos(x)
\]

#### Even Function Table Values

<table>
<thead>
<tr>
<th>x</th>
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</tr>
</thead>
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<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

#### Even Function Notation

If function f is an even function and \(f(0) = 0\), then what is \(f(x^2)\)?

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</tr>
<tr>
<td>-1</td>
<td>1</td>
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#### A List of Common Odd/Even Functions

<table>
<thead>
<tr>
<th>Odd Functions</th>
<th>Even Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = x^3)</td>
<td>(y = x^2)</td>
</tr>
<tr>
<td>(y = \sin(x))</td>
<td>(y = \cos(x))</td>
</tr>
<tr>
<td>(y = \tan(x))</td>
<td>(y = x)</td>
</tr>
<tr>
<td>(y = 1/x)</td>
<td>(y = \ln(x))</td>
</tr>
<tr>
<td>(y = x)</td>
<td></td>
</tr>
</tbody>
</table>

### Some Common Functions not Odd or Even

- \(y = \sqrt{x}\)
- \(y = 2^x\)